DWW-003-016304 Seat No. ________

M. Sc. (Mathematics) (Sem. - III) (CBCS) Examination

May / June – 2015

Maths. CMT - 3004 : Discrete Mathematics

Faculty Code : 003
Subject Code : 016304

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.

1 Answer any Seven **7×2=14**

(a) Define homomorphism of semigroups. Show that the homomorphic image of a monoid is a monoid.

(b) Define (i) a subsemigroup (ii) a submonoid. Illustrate them with examples.

(c) Define a lattice. If $(L,\leq)$ is a lattice, then verify that $(L,\leq^{-1})$ is also a lattice.

(d) Define a complemented lattice. Show that D_{2014} is complemented.

(e) Define a distributive lattice. Let $L=O$, where O is the collection of open sets in R. Verify that the lattice $(L,\subseteq)$ is distributive.

(f) Define the language of a phrase structure grammar.

(g) State Kleene's theorem.

(h) Define a Boolean Algebra. Give an example of a Boolean Algebra which contains exactly 128 elements.

(i) Define a machine congruence on a finite state machine.

(j) When do we say that two propositions p, q are logically equivalent?

2 Answer any Two **2×7=14**

(a) Define transitive closure of a relation R defined on a nonempty set A. If A contains exactly n elements, then prove that $\bigcup_{i=1}^{n} R^i$ is the transitive closure of R.

(b) (i) Let $L = \mathbf{N}$. Prove that L forms a distributive lattice under the divisibility relation.

(ii) Prove that the direct product of two bounded. lattices is also bounded.

(c) State the fundamental theorem of homomorphism of semigroups. Let A = {0, 1, 2}. Prove that $(N \cup \{0\}, +)$ is isomorphic to a quotient semigroup of A* .

3 (a) Let $G=(\{v_0, a, b\}, \{a, b\}, V_0 \mapsto)$ be a phrase structure grammar where the productions of G are given by **5**

(1) $v_0 \mapsto aav_0$, (2) $v_0 \mapsto a$, and 3. $v_0 \mapsto b$, Find $L(G)$.

(b) Find a Boolean expression for the function $f: B_4 \to B$ for which $S(f) = \{0000, 0010, 0101, 0111, 1101, 1111, 1000, 1010\}$ **5**

(c) Let $f_1, f_2 : B_n \to B$. Denoting the number of elements in a finite set. A by $|A|$, verify that $|S(f_1 \vee f_2)| = |S(f_1)| + |S(f_2)| - |S(f_1 \wedge f_2|$. 4

OR

3 (a) Construct a phrase structure grammar G with {a, b, c} as its set of terminal symbols such that $L(G) = \{a^n cb^n \mid n \geq 0\}$. 5

(b) Let $(L, \leq)$ be a lattice. Prove that the following statements are equivalent:

(i) For all $a,\ b,\ c\ \in L, a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$. 5

(ii) For all $a, b, c, \in L, a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$.

(c) Explain what do we mean by the quotient machine of a finite state machine M determined by a machine congruence R on M. 4

4 Answer any Two **2×7=14**

(a) Let M be a Moore machine with $S = \{s_0, s_1, s_2\}$, $I = \{0,1\}$, f_0 = the identity on S, $f_1(s_0) = s_1, f_1(s_1) = s_2$, and $f_1(s_2) = s_2$ and $T = \{s_2\}$. Find *L(M)*. Construct a type 3 phrase structure grammar G with I as its set of terminal symbols such that *L(G) = L(M)*.

(b) State and prove the Pumping lemma.

(c) Let $(L, \leq)$ be a lattice. If $(L, \leq)$ is not modular, then prove that L will contain a sublattice which is isomorphic to the pentagon lattice.

5 Answer any Two **2×7=14**

(a) Define (i) a conditional statement (ii) a biconditional statement (iii) a tautology. State and prove De Morgan's laws for logic.

(b) If M is a nondeterministic finte state machine with I as its input set, then prove that there exists a Moore machine M_1 with I as its input set such that $L(M)=L(M_1)$.

(c) Define a regular expression over a finite nonempty set A and also mention the rules for associating with each regular expression over A, a regular subset of A^*.

(d) Let $(A,\leq_A)$ be a lattice. If $f:\rightarrow.B$ is a bijection, then prove that there exists a relation $\leq_B$ defined on B in such a way that $(B,\leq_B)$ is also a lattice and $f:(A,\leq_A)\rightarrow(B,\leq_B)$ is an isomorphism of lattices.